

1. Calculate the potential energy of an electron in the $n=2$ state of the hydrogen atom.

$$\begin{aligned}
 E_p &= -\frac{Z^2 e^2}{4\pi \epsilon_0 n^2 a_0} \quad * \quad Z=1 \quad n=2 \\
 &= -\frac{e^2}{4\pi \epsilon_0 \cdot 4 a_0} \\
 &= \frac{(1.602 \times 10^{-19} \text{ C})^2}{16\pi \cdot (8.854 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2) (5.292 \times 10^{-11} \text{ m})} \\
 &= -10.9 \times 10^{-18} \text{ J}
 \end{aligned}$$

2. The Balmer spectrum of hydrogen (transitions into $n=2$) has a line of wavelength 656.7 nm. What is the wavelength of the corresponding line of deuterium? ($m_D = 3.3434 \times 10^{-27} \text{ kg}$; $m_H = 1.6727 \times 10^{-27} \text{ kg}$; $m_e = 9.1095 \times 10^{-31} \text{ kg}$).

$$\begin{aligned}
 \mu_H &= \frac{m_e \cdot m_H}{m_e + m_H} = 9.1046 \times 10^{-31} \text{ kg} \quad \mu_D = \frac{m_e \cdot m_D}{m_e + m_D} = 9.1070 \times 10^{-31} \text{ kg} \\
 \frac{1}{\lambda} &= \frac{Z^2 e^2}{8\pi \epsilon_0 a_0 h c} \left(\frac{1}{n_2} - \frac{1}{n_1} \right); \quad a_0 = \frac{h^2 \epsilon_0}{\pi e^2 \mu} \Rightarrow \frac{1}{\lambda} \propto \mu \\
 \Rightarrow \frac{\lambda(D)}{\lambda(H)} &= \frac{\mu_H}{\mu_D} \Rightarrow \lambda_D = \frac{\mu_H}{\mu_D} \cdot 656.7 \text{ nm} \Rightarrow \lambda \propto \frac{1}{\mu} \\
 &= 656.30 \text{ nm}
 \end{aligned}$$

3. In the microwave spectrum of $^{12}\text{C}^{16}\text{O}$ the separation between lines ($\Delta\nu$) has been measured to be 115270 MHz. Calculate the interatomic distance.

$$\begin{aligned}
 \nu &= 2(l+1)B \Rightarrow \Delta\nu = 2B \Rightarrow B = 5.7635 \times 10^4 \text{ s}^{-1} \\
 B &= \frac{h}{8\pi^2 I} \Rightarrow I = \frac{h}{8\pi^2 B} = 1.456 \times 10^{-46} \text{ kg m}^2 \\
 I &= \mu d^2 \Rightarrow d = \sqrt{\frac{I}{\mu}} \quad \mu = \frac{12 \times 16}{28 \times 6.022 \times 10^{23}} \text{ kg} \\
 &= 1.131 \times 10^{-10} \text{ m} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} = 1.1385 \times 10^{-26} \text{ kg}
 \end{aligned}$$