CHEM 3423 001 Spring 2024 **Name:** KEY
ID:

Worksheet #4 (Maximum number of points you can get is **3 pts**)

1. For H_2 gas at 400 K, calculate the ratio of the fraction of molecules that have the speed $u_2 = 3\,u_1$ to the fraction that have speed u_1. Assume that $u_1 = \tilde{u}$ (the average speed of molecules at this temperature).

$$\frac{N(u)}{N} = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-mu^2/2k_BT} \cdot u^2 du$$

$$\Rightarrow \frac{N(u_2)}{N(u_1)} = \frac{u_2^2 \exp\left(-\frac{mu_2^2}{2k_BT}\right)}{u_1^2 \exp\left(-\frac{mu_1^2}{2k_BT}\right)} = 9 \exp\left(-\frac{m}{2k_BT}(9-1)u_1^2\right)$$

$$= 9 \cdot \exp\left(-\frac{m}{2k_BT} \cdot 8u_1^2\right) \qquad u_1 = \tilde{u} = \sqrt{\frac{8k_BT}{\pi m}}$$

$$= 9 \cdot \exp\left(-\frac{8m}{2k_BT} \cdot \frac{8k_BT}{\pi m}\right) = 9 \cdot \exp\left(-\frac{32}{\pi}\right)$$

$$= 9 \cdot \exp(-10.2) = 9 \cdot 0.0000377 = 0.000339 = 334.10^{-4}$$

2. Hydrogen gas has a molecular collision diameter of 0.258 nm. Calculate the mean free path of hydrogen at 300 K and 200 kPa. ($k_B = R/L = 1.381 \times 10^{-23}$ J/K)

$$\lambda = \frac{V}{\sqrt{2}\,\pi d_A^2 N} \quad | \quad \frac{V}{N} = \frac{RT}{LP} = \frac{k_BT}{P}$$

$$\Rightarrow \lambda = \frac{k_BT}{\sqrt{2}\,\pi d_A^2 P} = 7.06 \cdot 10^{-8}\ m$$

3. (Bonus question) Write down the barometric distribution law. Suppose the composition of a gas remains the same, is the air pressure on a hot summer day higher or lower than on a cold winter day?

$$P = P_0 \cdot e^{-Mgz/RT} \qquad T \uparrow \Rightarrow P \uparrow$$

higher on summer day